WYNEDUCATION

Mathematical Methods
Written Exam 2
Multiple Choice Question Booklet

Section	Number of Questions	Number of Marks
A	20	20

Materials Permitted

- Approved CAS Calculator
- Formula Sheet

Recommended Time (Section A)

- 30 minutes

Solutions

Visit @wyneducation on YouTube for detailed solutions

- Video Title: "2024 Methods Trial Exam 2 MC Solutions"

SECTION A – Multiple Choice Questions

1. Given that $f(x) = 2x + 1$ and $g(x) = x^2 - 3$, what is the value of $f\big(g(2)\big)$?

   A. 1

   B. 3

   C. 5

   D. 7

2. Consider the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x + a_0$.

   What is the maximum number of turning points the graph of the polynomial can have?

   A. n

   B. $n - 1$

   C. $n - 2$

   D. $n - 3$

3. For the graph of $f(x) = 2\sin(x) - b$. The values of b that ensure $f(x) > 0$ for all x are

   A. $b > 0$

   B. $b < 0$

   C. $b > -2$

   D. $b < -2$

4. The set of ordered pairs $\{(-4, -1), (-4,1), (6,8), (6, -3)\}$ is

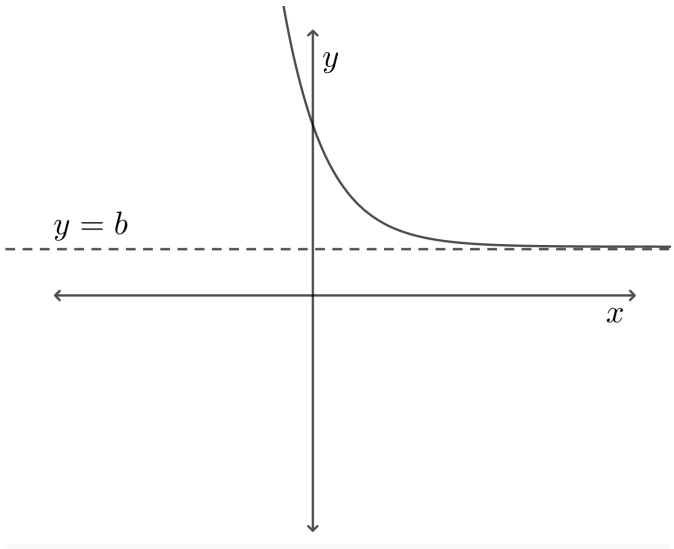
   A. One-to-one

   B. Many-to-one

   C. One-to-many

   D. Many-to-many

5. If $a > 0$ and $b < 0$. The graph below could be represented by which of these rules?



A. $y = ae^{-x} + b$

B. $y = -ae^{x} - b$

C. $y = -ae^{-x} + b$

D. $y = ae^{-x} - b$

6. Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$. The domain of $f(x) \cdot g(x)$ is

A. $x \in [0,2) \cup (2, \infty)$

B. $x \in [0, \infty)$

C. $x \in (-\infty, 2) \cup (2, \infty)$

D. $x \in (-\infty, \infty)$

7. A fair coin is flipped 10 times. What is the probability of getting exactly 7 heads?

A. $\binom{10}{3} \times \left(\frac{1}{2}\right)^{7}$

B. $\binom{10}{7} \times \left(\frac{1}{2}\right)^{7}$

C. $\binom{10}{3} \times \left(\frac{1}{2}\right)^{10}$

D. $\binom{7}{3} \times \left(\frac{1}{2}\right)^{10}$

8. Let X and Y be discrete random variables with probability distributions shown below. For what values of k is the mean of X (μ_X) greater than the mean of $Y(\mu_Y)$?

X	1	2	3	4
$\Pr(X = x)$	$\frac{1}{4}$	$\frac{3k}{4}$	$\frac{1-k}{4}$	$\frac{1}{4}$

Y	1	2	3	4
$\Pr(Y = y)$	$\frac{1}{4}$	$\frac{k}{4}$	$\frac{2(1-k)}{4}$	$\frac{1}{4}$

A. $k > \frac{1}{2}$

B. $k > \frac{5}{8}$

C. $k > \frac{1}{3}$

D. $k > \frac{3}{7}$

9. For two events the $\Pr(A) = 0.35$ and $\Pr(B) = 0.45$ and the $\Pr(A|B) = 0.22$. Which of the following statements **must** be true?

A. A and B are independent events

B. A and B are mutually exclusive events

C. A and B are independent events because $\Pr(A|B) \neq \Pr(A)$

D. $\Pr(A \cap B) = \Pr(A|B) \cdot \Pr(B)$

10. If $f(x)$ is a probability density function, the value of k is.

$$f(x) = \begin{cases} k(1-x)^2, -1 \leq x \leq 1 \\ 0, \quad otherwise \end{cases}$$

A. $\frac{2}{3}$

B. 1

C. $\frac{3}{2}$

D. $\frac{3}{8}$

11. If $f(x) = \frac{2x+1}{x-5}$ and $g(x) = \sqrt{x-a}$, $f(g(x))$ has a vertical asymptote at x equals

A. a^2

B. $25 + a$

C. $25 - a$

D. $25 - a^2$

12. $\int_0^{3k} f(\frac{x}{k})\, dx$ is equal to

A. $k \int_0^3 f(x)\ dx$

B. $\frac{1}{k} \int_{-3k}^{3k} f(x)\ dx$

C. $k^2 \int_0^{3k} f(x)\ dx$

D. $\int_0^6 f(x)\ dx$

13. A bag contains three red balls and five green balls. Three balls are chosen at random **without** replacement. What is the probability that at least one ball is red?

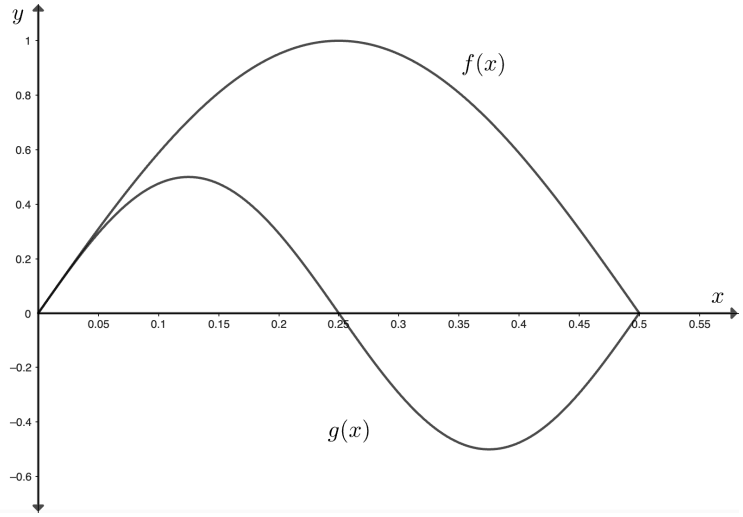
A. $\frac{15}{28}$

B. $\frac{5}{8}$

C. $\frac{3}{8}$

D. $\frac{23}{28}$

14. Parts of the graphs of $f(x) = \sin(2\pi x)$ and $g(x) = \frac{1}{2}\sin(4\pi x)$ are shown below



The pseudocode below describes an algorithm to find the value of x where the functions are furthest apart

```
Define:    f(x)=sin(2*pi*x)
           g(x)=1/2*sin(4*pi*x)
           h(x)=|f(x)-g(x)|

Set max_distance=0
Set best_x=0
Set step_size=0.05

While x <= 0.5
      current_distance = h(x)

     if current_distance > max_distance then
          set max_distance ← current_distance
          set best_x ← x

     x ← x + step_size

Return best_x, max_distance
```

The pseudocode will return the following value for `best_x`

A. 0.30

B. 0.33

C. 0.35

D. 0.40

15. If α and β are solutions to the quadratic equation $ax^2 + bx + c = 0$. The value of $\alpha + \beta$ and $\alpha\beta$ respectively are

A. $\alpha + \beta = \frac{b}{a}, \alpha\beta = \frac{-c}{a}$

B. $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{-c}{a}$

C. $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

D. $\alpha + \beta = \frac{-b}{2a}, \alpha\beta = \frac{c}{a}$

16. Let $f: [0, a] \to \mathbb{R}, f(x) = x log_e\left(\frac{a}{x}\right)$. The maximum value of $f(x)$ is

A. e

B. $\frac{a}{e}$

C. $a log_e(\frac{1}{e})$

D. a

17. Consider the hybrid function

$$h(x) = \begin{cases} kx + 1, x \geq a \\ x^2, \quad x < a \end{cases} \quad where\ a \in \mathbb{R}\backslash\{0\}$$

The value of k such that $h(x)$ is continuous at $x = a$ is

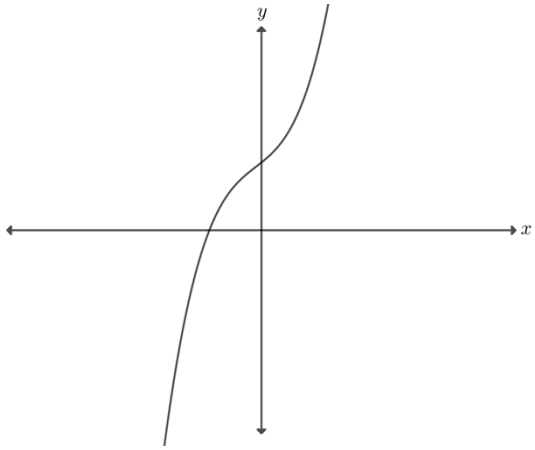
A. $k = a - 1$

B. $k = \frac{a^2 - 1}{a}$

C. $k = 2a$

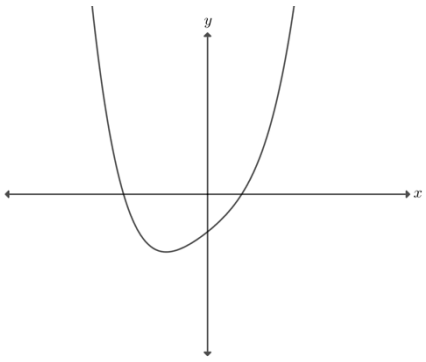
D. $k = 1 - a^2$

18. The graph of $y = f'(x)$ is shown below.

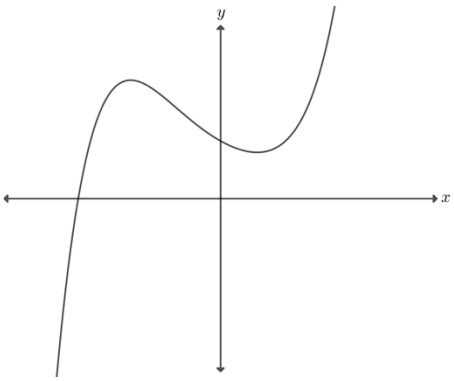


The graph of $y = f(x)$, the first anti-derivative of $f'(x)$ could be

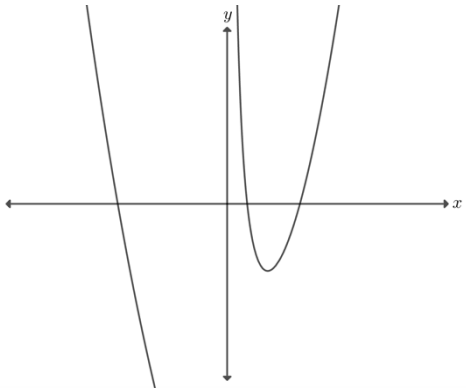
A.



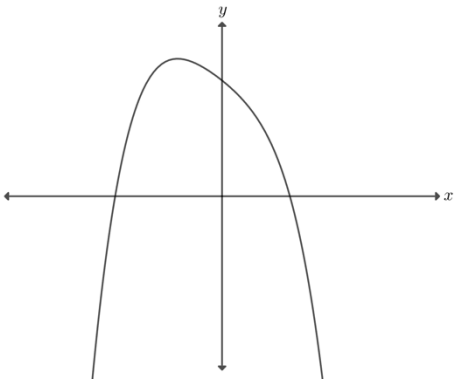
B.



C.



D.



19. Let $f(x) = e^x$ and $g(x) = px$. The values of p such that there is one solution are

A. $p \in (-\infty, 0) \cup (0, e)$

B. $p \in \{0\} \cup (e, \infty)$

C. $p \in (0, e)$

D. $p \in (-\infty, 0) \cup \{e\}$

20. A researcher is studying the average height of adult males in a particular city. After collecting data from an initial sample of 30 men, a 95% confidence interval for the average height is calculated. The researcher then decides to collect a larger sample of 150 men from the same population.

    What will happen to the width of the 95% confidence interval for the average height as the sample size increases? Assume the mean and standard deviation remain the same.

    A. The width of the confidence interval will increase.

    B. The width of the confidence interval will decrease.

    C. The width of the confidence interval will remain the same.

    D. The confidence interval will become invalid.